Name of Subject :		Marks obtained :	
Sign. of Supervisor :		Sign. of Sub. Examiner :	
Q. No. 1	Q. No. 2	Q. No. 3	Total Marks

Supplied By: SNJB

000567

3. Overview of Transformations.

Transformation

A graphics system must allow the user to change the way object appears.

We can change the size of object, its position on the screen, or its orientation. It is much more sensible to make changes later to the objects original basic description. Implementing such a change is called transformation

Basically transformation means making changes or allowing modifications to the picture

Matrix Representation

matrix is two-dimensional array of number

$$\begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{vmatrix} \quad \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix} \quad \begin{vmatrix} 0 & -1 & 3 \\ 0 & 1 & 2 \\ 5 & 4 & 2 \end{vmatrix}$$

The element in matrix are identified by specifying the row & column number.

eg:-

$$A = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} \quad B = \begin{vmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{vmatrix}$$

$$C = \begin{vmatrix} 48 & 60 \\ 57 & 72 \\ 66 & 84 \end{vmatrix}$$

Identify Matrix

The square matrix which has all the elements equal to 0 except the elements of diagonal, which are all 1 is called identity matrix

$$|1| \quad \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The identity matrix is denoted as I
&
$A = A * I$

Transformation Conventions

usually two conventions are used to represent data & to perform transformations with matrix multiplication

: Post multiplication transformation
Pre ———"———

eqn represents a general transformation
$[X'] = [X][T]$

where
T represent any transformation such as rotation, scaling or translation
since in this eqn position vector are represented as row matrices, the transformation matrix appears after the data or position vector matrix. This is post-multiplication transformation.

If we choose to represent the position vector as column matrix we have

$$[X'] = [T]^T [X]^T$$

where $[T]^T$ & $[X]^T$ are the transpose of the corresponding matrices.

Using homogeneous co-ordinates for the positive rotation by an angle of an object about the origin (z-axis) using post multiplication transformation

$$[x' \; y' \; 1] = [x \; y \; 1] \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

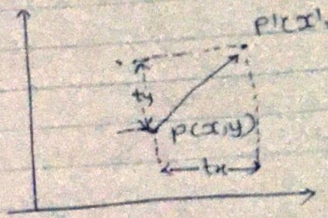
However, using pre-multiplication transformation, we can perform same transformation using

$$[x' \; y' \; 1] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

* Two-dimensional Translation

① Translation

- Translation is a process of changing the position of an object in a straight line path from one co-ordinate locatn to another.
- We can translate a two dimensional point by adding translation distances t_x & t_y, to the original co-ordinates position (x, y) to move the point to new position (x', y')



$x' = x + t_x$

$y' = y + t_y$

The translation distance pair (t_x, t_y) is called translation vector or shift vector.

- a single matrix eqn by using column vectors to represent co-ordinate positions & the translation vector.

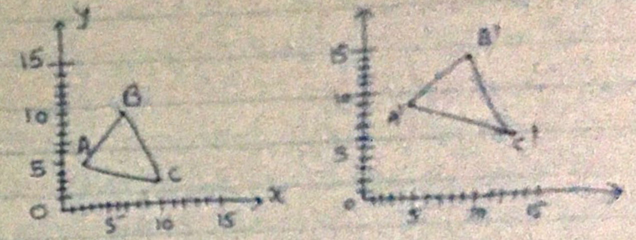
$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

This allows us to write the two dimensional translation eqn in matrix form

$$P' = P + T$$

eg ① Translate a polygon with co-ordinates A(2,5), B(7,10), C(10,2) by 3 units in x direction & 4 units in y direction

→



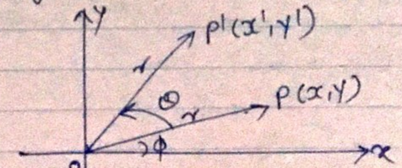
$$A' = A + T = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

$$B' = B + T = \begin{bmatrix} 7 \\ 10 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

$$C' = C + T = \begin{bmatrix} 10 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 13 \\ 6 \end{bmatrix}$$

② Rotation

- A two dimensional rotation is applied to the an object by repositioning it along a circular path in the xy plane
- To generate a rotation, we specify a rotation angle θ & the position of the rotation point about which the object is to be rotated.



Here,

r is constant distance of the point from origin

angle ϕ is original angular position of the point from the horizontal, & θ is rotation angle.

using standard trigonometric eqⁿ, we can express the transformed co-ordinates in terms of angles θ & ϕ

$$\left.\begin{aligned} x' &= r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ y' &= r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{aligned}\right\} \quad (1)$$

The original co-ordinates of the point in polar co-ordinates are given below

$$\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned} \quad \text{———} \quad (2)$$

substituting (2) into (1)

we get the transformation eqⁿ for rotating a point (x,y) through angle θ about origin as:

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta.$$

The above eqⁿ can represent in matrix form

$$[x' \; y'] = [x \; y] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$P' = P \cdot R$$

where,

R is rotation matrix

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

positive values for the rotation angle define counterclockwise rotations about the rotation point

negative values rotate objects in the clockwise sense

for negative value of θ is for clockwise rotation,

the rotation matrix becomes

$$R = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\because \cos\theta = \cos(-\theta)$

$-\sin\theta = \sin(-\theta)$

Q. A point (4,3) is rotated counterclockwise by angle of 45°. find the rotation matrix & the resultant point

$\rightarrow$

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$P' = P \cdot R$

$$= \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}$$

③ Scaling

Changes the size of an object

This operation can be carried out for polygons by multiplying the co-ordinate values (x, y) of each vertex by scaling factors S_x & S_y to produce the transformed co-ordinates (x', y')

$x' = x \cdot S_x$

$y' = y \cdot S_y$

Scaling factor S_x scales object in x direction

Scaling factor S_y scales object in y direction.

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$= \begin{bmatrix} x \cdot S_x & y \cdot S_y \end{bmatrix}$$

$$= P \cdot S$$

S.N.J.B.'s
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CLASS TEST I / II (201 - 201)

Course & Semester :		Roll No.:	Date : / /201
Name of Subject :			Marks obtained :
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- Any positive numeric values are valid for scaling factors S_x & S_y. Values less than 1 reduce the size of the objects & values greater than 1 produce enlarged object.
- For both S_x & S_y values equal to 1, the size of object does not change
- To get uniform scaling it is necessary to assign same value for S_x & S_y. Unequal value for S_x & S_y result in differential scaling

Q, Scale polygon with co-ordinates A(2,5), B(7,10) & C(10,2) by two units in x direction & two units in y direction.

→ Here $S_x = 2$ & $S_y = 2$

$$\therefore S = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

The object matrix is $A = \begin{bmatrix} 2 & 5 \\ 7 & 10 \\ 10 & 2 \end{bmatrix}$

$$A' = P \cdot S = \begin{bmatrix} 2 & 5 \\ 7 & 10 \\ 10 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 10 \\ 14 & 20 \\ 20 & 4 \end{bmatrix}$$

Homogeneous co-ordinates for Translation

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{bmatrix}$$

Therefore, we have

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{bmatrix}$$

$$= [x + tx \ y + ty \ 1]$$

Homogeneous Co-ordinates for rotation

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore,

$$[x' \ y' \ 1] = [x \cdot y \ 1] \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [x\cos\theta - y\sin\theta \ \ x\sin\theta + y\cos\theta \ \ 1]$$

Homogeneous Co-ordinates for Scalling

$$S = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, $[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= [x.Sx \ \ y.Sy \ \ 1]$$

Q.1 Given 3X3 homogeneous co-ordinate transformation matrix for each of the foll^n translation
1) shift the image to right 3 unit
2) shift the image ~~d~~ up ~~l~~ 2 unit
3) Move the image down 1/2 unit & right 1 unit
4) Move the image down 2/3 unit & left 4 unit

→ $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{bmatrix}$

Here $tx = 3$, $ty = 0$

$$T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

Here $tx = 0$, $ty = 2$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

Here, $tx = 1$, $ty = -0.5$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -0.5 & 1 \end{bmatrix}$$

Here, $tx = -4$, $ty = -0.66$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -0.66 & 1 \end{bmatrix}$$

Q2 Find the transformation matrix that transforms the given square ABCD to half its size with center still remaining at the same position. The co-ordinates of the square are A(1,1), B(3,1) C(3,3), D(1,3) & center at (2,2). Also find resultant co-ordinate of square

→

$$S = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Steps ① translate square so that its center coincide with origin
② scale the square with respect to the origin
③ Translate the square back to the original position

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$T_1' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix}$$

$$\therefore T_1 \cdot S \cdot T = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 3 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & 1.5 & 1 \\ 2.5 & 1.5 & 1 \\ 2.5 & 2.5 & 1 \\ 1.5 & 2.5 & 1 \end{bmatrix}$$

Q3 Find the transformation of triangle A(1,0) B(0,1), C(1,1) by a) Rotating 45° about the origin & then translating one unit in x & y direction. b) Translating one unit in x & y direction & then rotating 45° about the origin.

→

$$R = \begin{bmatrix} \cos 45° & \sin 45° & 0 \\ -\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$a)\ R \cdot T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2}+1 & 1/\sqrt{2}+1 & 1 \\ -1/\sqrt{2}+1 & 1/\sqrt{2}+1 & 1 \\ 1 & \sqrt{2}+1 & 1 \end{bmatrix}$$

b.) $T \cdot R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{bmatrix}$

$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1/\sqrt{2} & 3/\sqrt{2} & 1 \\ -1/\sqrt{2} & 3/\sqrt{2} & 1 \\ 0 & 2/\sqrt{2} & 1 \end{bmatrix}$

Composite Transformations :-

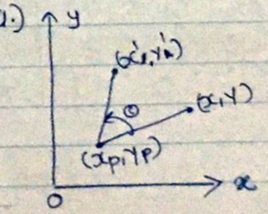
The basic purpose of composing transformation is to gain efficiency by applying a single composed transformation to point, rather than applying series of transformations.

Rotation about an Arbitrary Point

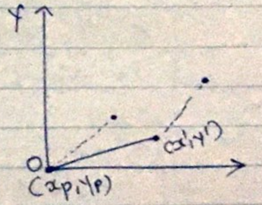
- To rotate an object about an arbitrary point (x_p, y_p)

steps: ① Translate point (x_p, y_p) to the origin
② Rotate it about the origin &
③ finally, translate the center of rotation back to where it belongs.

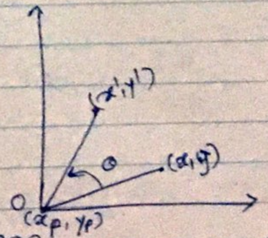
a.)



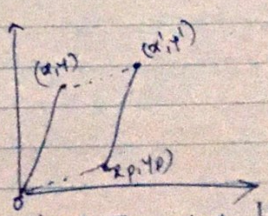
Rotation about an arbitrary point



step1: Translate point (x_p, y_p) to the origin



step2: Rotate it about the origin



step3: Translate back to the original position

Translate matrix to move point (x_p, y_p) to the origin is

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{bmatrix}$$

The rotation matrix for counterclockwise rotation of point about the origin

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The translation matrix to move the center point back to its original position is given as

$$T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$$

$$\therefore T_1 \cdot R \cdot T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_p\cos\theta + y_p\sin\theta + x_p & -x_p\sin\theta - y_p\cos\theta + y_p & 1 \end{bmatrix}$$

Q.1 A polygon has 4 vertices located at A(20,10) B(60,10) C(60,30) D(20,30). Calculate the vertices after applying a transformation matrix to double the size of polygon with point A located on same place

$\longrightarrow$

$$T = T_1 \cdot S \cdot T_2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -20 & -10 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 20 & 10 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -20 & -10 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 20 & 10 & 1 \\ 60 & 10 & 1 \\ 60 & 30 & 1 \\ 20 & 30 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -20 & -10 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 10 & 1 \\ 100 & 10 & 1 \\ 100 & 50 & 1 \\ 20 & 50 & 1 \end{bmatrix}$$

Q.2 Perform a 45° rotation of triangle A(0,0) B(1,1), C(5,2) i) About the origin ii) about P(-1,-1)

$\longrightarrow$ assume that rotation clockwise

i) $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ where $\theta = 45°$

$$R = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ \sqrt{2} & 0 & 1 \\ 7/\sqrt{2} & -3/\sqrt{2} & 1 \end{bmatrix}$$

After rotation,

$A = (0,0)$ $B = (\sqrt{2}, 0)$ $C(7/\sqrt{2}, -3/\sqrt{2})$

ii) About point $(-1,-1)$

$$T_1 \cdot R\, T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ +1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45 & 0 \\ \sin 45 & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & \phi \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \sqrt{2}-1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \sqrt{2}-1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}-1 & -1 & 1 \\ \sqrt{2}+\sqrt{2}-1 & -1 & 1 \\ 9/\sqrt{2}-1 & -3/\sqrt{2}-1 & 1 \end{bmatrix}$$

③ Consider the square $A(1,0)$, $B(0,0)$, $C(0,1)$, $D(1,1)$. Rotate the square by 45° anticlock wise direction followed by reflection about x-axis

$\longrightarrow$

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos\theta & -\sin\theta & 0 \\ -\sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where, $\theta = 45^\circ$

$$T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 0 & -\sqrt{2} & 1 \end{bmatrix}$$

* Reflection

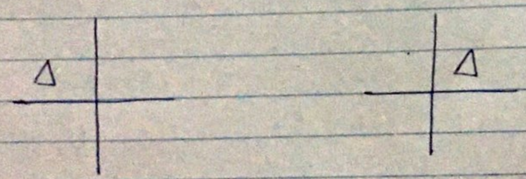
It is transformation that produces a mirror image of an object relative to an axis of reflection.

We can choose an axis of reflection in xy plane or perpendicular to the xy plane

1) Reflection of about Y-axis

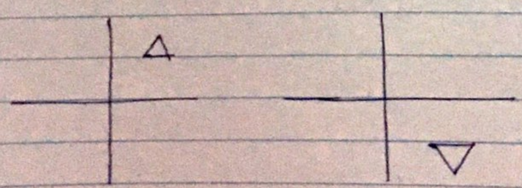
$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Transformation matrix

original image　　　Reflected image



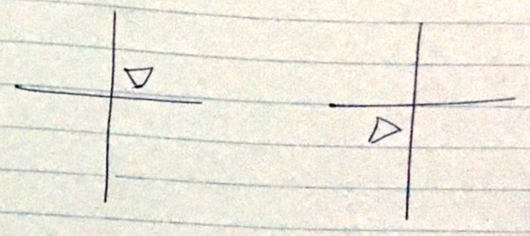
2) Reflection about X-axis

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



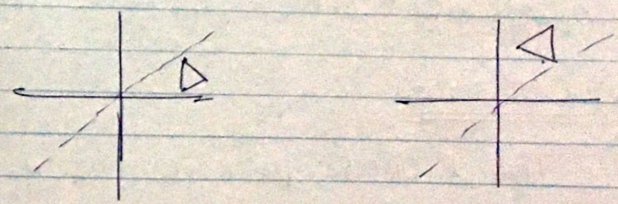
Reflection about origin

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



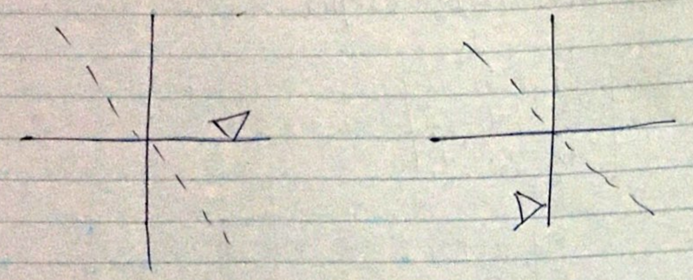
Reflection about line $y = x$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflection about line $y = -x$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



* Shear

A transformation that slants the shapes of an object is called shear transformation

One shift x-co-ordinate values
& other shifts y co-ordinate values
However, in both the case only one co-ordinate (x or y) changes its co-ordinate & other preserves its value

1) X shear

The x-shear preserves the y-coordinates but changes in x values which causes vertical lines to tilt right or left

$$X_{sh} = \begin{bmatrix} 1 & 0 & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = x + Sh_x \cdot y$. & $y' = y$

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Supplied By : SNJB

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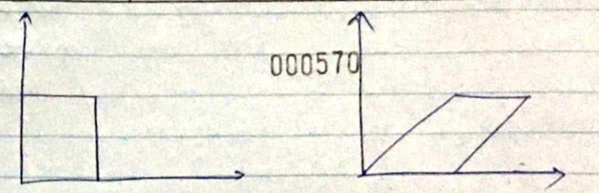
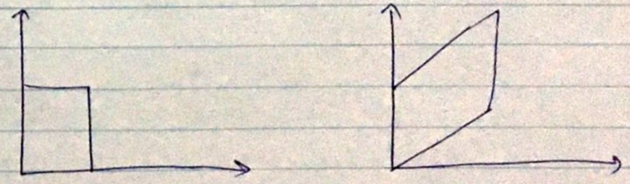


original object   object after x shear

2) Y shear

The y shear preserves the x co-ordinates, but changes the y values which causes horizontal lines to transform into lines which slope up or down.



original object   object after y shear

$$Y_{sh} = \begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = x$

$y' = y + Sh_y \cdot x$

* Shearing Relative to other reference Line

We apply x shear & y shear transformations relatives to other reference lines. In x shear transformation we can use y reference line & in y shear we can use x reference line

Shear with y reference line

$$= \begin{bmatrix} 1 & 0 & 0 \\ shx & 1 & 0 \\ -shx.y_{ref} & 0 & 1 \end{bmatrix}$$

Shear with x reference line

$$\begin{bmatrix} 1 & shy & 0 \\ 0 & 1 & 0 \\ 0 & -shy.x_{ref} & 1 \end{bmatrix}$$

Q. Apply the shearing transformation to square A(0,0), B(1,0), C(1,1) & D(0,1) as given below

① Shear parameter value of 0.5 relative to the line $y_{ref} = -1$

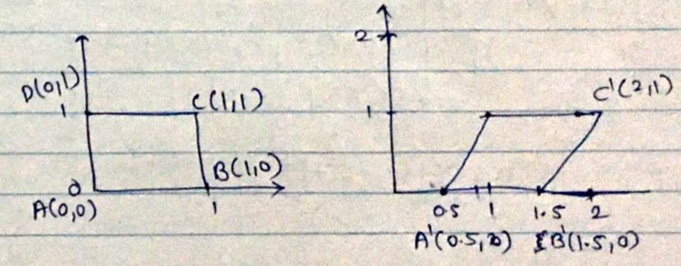
② Shear parameter value of 0.5 relative to the line $x_{ref} = -1$

→ $shx = 0.5 \quad y_{ref} = -1$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ shx & 1 & 0 \\ -shx.y_{ref} & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 1 \\ 1.5 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

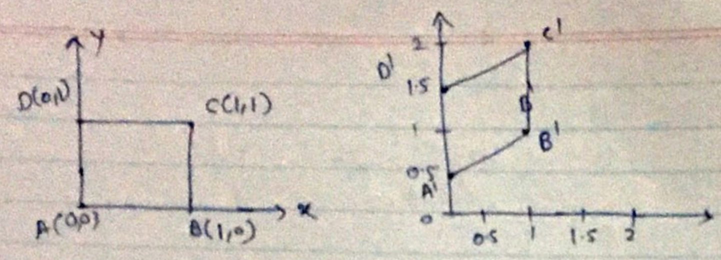


b) Here,

$shy = 0.5$ & $x = -1$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \begin{bmatrix} 1 & shy & 0 \\ 0 & 1 & 0 \\ 0 & -shy.x_{ref} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1.5 & 1 \end{bmatrix}$$



Three Dimensional Transformations

Translation

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$P' = P \cdot T$$

$$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$P' = [x+t_x \;\; y+t_y \;\; z+t_z \;\; 1]$$

Scaling :-

$$S = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = P \cdot S$$

$$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [x \cdot S_x \;\; y S_y \cdot \;\; z S_z \;\; 1]$$

Rotations

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ ~~for~~

Rotation about x axis

The positive value of angle θ indicates counterclockwise rotation. for clockwise rotation value of angle θ is negative

$$R_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation about y axis

Types of Projections

After converting the description of objects from would co-ordinates to viewing co-ordinates, we can project the three dimensional object onto the two dimensional view plane

There are two basic ways of projecting objects onto the view plane

1) Parallel projection
2) Perspective projection

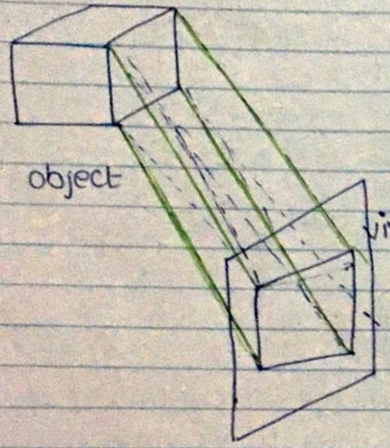
1) Parallel Projection

z co-ordinate is discarded & parallel

from each vertex on the object are extended until they intersect the view plan. The point of intersection is the projection of the vertex.

We connect the projected vertices by line segments which correspond to connections on the original object.

It preserves relative proportions of object but doesnot produce the realistic views.

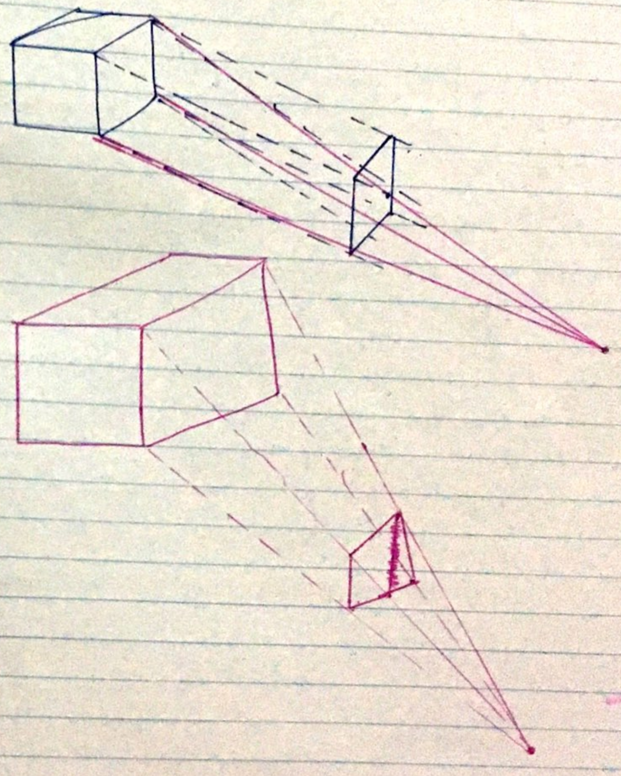


~~Persp~~ Perspective Projection

produces realistic view but does not preserve relative proportions.

In this, the lines of projection are not parallel. Instead, they all converage in single point called the center of projection or projection reference point

The object positions are transformed to the view plane along these coverged projection lines & the projected view of an object is determined by calculating the intersection of the coverged projection line with the view plan



Types of Parallel projection

depending on the relation between the direction of projection & normal to the view plane.

When the direction of the projection is normal (perpendicular) to the view plane, we have an orthographic parallel projection. Otherwise, we have an oblique parallel projection.

Orthographic Projections are further classified as axonometric orthographic & multiview orthographic projection.

The orthographic Projection ~~are further classified as axonometric~~ can display more than one face of an object. Such as orthographic projection is axonometric orthographic projection

axonometric projections are three types-

1) Isometric - All three principle axes are foreshortended equally
2) Dimetric - Two principle axes are foreshortened equally
3) Trimetric - All three principle axes are foreshortened unequally

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- The Oblique Projection are further classified as cavalier & cabinet projections

- For the cavalier projection, the direction projection makes a 45° angle with the view plane.
- When the direction of projection makes an angle of arctan(2) = 63.4° with the view plane, the resulting view is called cabinet projection

Types of Perspective Projection

- The perspective projection of any set of ||el lines that are not ||el to projection plane converge to vanishing point.

- The vanishing point for any set of lines that are ||el to one of the three principle axes of object is referred to as principle vanishing point or axis vanishing point

- There are at most three such points, corresponding to the number of principle axes cut by the projection plane. The perspective projection classifired according to no of principle vanishing points : one-point, two point, three points.